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Heat transfer from a rigid sphere in a nonuniform flow and temperature field

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Abstract—The energy equation of a rigid sphere in a viscous fluid subject to an unsteady flow and temperature field is developed. A perturbation method is used to derive the heat transfer from a rigid sphere at low Peclet numbers. Thus, the temperature field is decomposed into the undisturbed field and the disturbance due to the presence of the sphere. A symmetry relationship is used to yield the rate of heat transfer due to the disturbance in the Laplace domain. The transformation of the rate of heat transfer in the time domain yields a history integral, which combines the effects of all past temperature changes of the sphere. This history integral in the energy equation is analogous to the history force (or Basset force) in the equation of motion of the sphere. By the heat–momentum transfer analogy, it is anticipated that the history term will play an important role in liquid–solid heat or mass transfer and that, depending on the frequency of the fluid temperature domain, it may account for up to 25% of the instantaneous heat transfer.

INTRODUCTION

THE EQUATION for the creeping motion of a spherical particle was derived more than one hundred years ago as an exact solution of the Navier–Stokes equations in spherical coordinates by Boussinesq [1] and Basset [2]. Several higher order corrections to the Boussinesq–Basset equations have been proposed since then; one of the most recent ones was done by Maxey and Riley [3] who derived the complete equation of motion of a particle in an unsteady flow field. One of the salient features of the equation of motion of the particle is the appearance of the history integral term

$$6\alpha^2\sqrt{(\mu\rho_f\pi)}\int_0^t \frac{\dfrac{d}{d\tau}[\mathbf{V}(\tau)-\mathbf{u}[\mathbf{Y}(\tau),\tau]-\frac{1}{6}\alpha^2\nabla^2\mathbf{u}\,|_{\mathbf{Y}(\tau)}]}{\sqrt{(t-\tau)}}\,d\tau$$

which includes all the past accelerations of the sphere in the unsteady velocity field. This term augments the steady-state drag on the sphere. This integral term is sometimes called the Basset term or the history term [4] and was observed [5] to account for as much as 25% of the instantaneous drag force on the sphere.

Regarding the heat transfer equation from a sphere, it is usually given for the steady-state conditions in terms of an average heat transfer coefficient and the temperature difference between the sphere and the surrounding fluid. This results to a simpler form for the energy equation for the sphere [4] which has been used in many engineering applications. It has been observed, however [4, 6], that fluid oscillations improve the heat and mass transfer rates in industrial equipment. Part of this improvement is probably due to the increased holdup of particles in the column and to the observed break-up of bubbles and droplets. There is another part, however, which has to do with the nature of the energy equation of the sphere in an unsteady flow and temperature field. This part is analogous to the history term of the equation of motion of the sphere and includes the effects of the all previous temperature changes of the sphere and the surrounding fluid. An allusion to this term for the unsteady conduction regime appears in Carslaw and Jaeger [7]: the error function term, which includes the history of previous temperature changes appears in several solutions for the heat conduction from a sphere or a cylinder. Except for this allusion, however, there is no other evidence of the history term in the energy equation of a spherical particle in a viscous fluid.

The energy equation of a sphere in unsteady temperature and velocity fields is developed in the present work. The temperature field is decomposed into an undisturbed component and the disturbance caused by the presence of the sphere. The total rate of heat transfer due to both components is obtained first in the Laplace domain and then by a transformation in the time domain. Hence, the complete energy equation for a sphere in a time-varying temperature field is developed. The energy equation includes a term, which is analogous to the history term of the equation of motion.

ENERGY EQUATIONS FOR THE FLUID AND THE SPHERE

We consider a fluid moving with velocity v_i relative to a fixed coordinate system $0x_1x_2x_3$ and a sphere moving inside this fluid with velocity $V_i(t)$ relative to the same coordinate system. The temperature of the

NOMENCLATURE

A, B, C	functions defined by equations (22b)–(22d)
c	specific heat capacity
F, G	functions in Laplace space
k	conductivity
m	mass
q	heat flux
$\mathbf{n}$	outward vector
Pe	Peclet number
Pr	Prandtl number
Q	rate of heat transfer
r	radial coordinate
s	transformed variable in Laplace space
t	time
T	temperature
$\hat{U}$	characteristic relative velocity of the particle
v	fluid velocity in x coordinates
V	velocity of sphere
w	fluid velocity in z coordinates
x	fixed coordinate system
Y	coordinate of the sphere in x system
z	coordinates moving with the sphere.

Greek symbols

α	radius of sphere
η	ratio of properties
θ	characteristic temperature
λ	length scale defined in equation (25)
μ	dynamic viscosity
ν	kinematic viscosity
π	3.14159
ρ	density
τ	parameter with units of time.

Superscripts

0	pertaining to unperturbed field
1	pertaining to perturbation
∞	evaluated far from the sphere
$\hat{}$	dimensionless
$\bar{}$	Laplace transform
$'$	pertaining to the auxiliary field used in equation (21c).

Subscripts

f	fluid
i, j, k	indices
s	sphere.

fluid is non-uniform in time and of course is a space function too. Heat may be transferred from the sphere to the fluid through conduction at the spherical boundary. The energy equation for the fluid in the Eulerian coordinate system $0x_1x_2x_3$ may be written as follows:

$$\rho_f c_f \left(\frac{\partial T_f}{\partial t} + v_i \frac{\partial T_f}{\partial x_i}\right) = k_f \frac{\partial^2 T_f}{\partial x_i \partial x_i}, \qquad (1)$$

where ρ is the density, c is the specific heat capacity, k the thermal conductivity and T the temperature of the fluid; v_i is the component of the fluid velocity in the ith direction (as modified by the presence of the sphere) and t the time. The subscript f denotes fluid properties and related variables, while the subscript s will be used to denote properties and variables related to the sphere. It is assumed that the flow is slow enough for the dissipation term $\mu\Phi$ to be neglected in the energy equation for the fluid. It is also assumed that the sphere is rigid. Hence, its velocity satisfies the appropriate equation of motion, but is a function of time only.

For the calculations involving the sphere it is convenient to use a Lagrangian coordinate system moving with the sphere, $0z_1z_2z_3$. If the coordinates of the center of the sphere are $Y_i(t)$ with respect to the Eulerian system $0x_1x_2x_3$ and the sphere moves with velocity $V_i(t)$ then the new system of coordinates is defined as:

$$\mathbf{z} = \mathbf{x} - \mathbf{Y}(t). \qquad (2)$$

Hence, the functional relationship for the temperature of the fluid and the relative velocity of the fluid with respect to the sphere become:

$$T_f(\mathbf{x}, t) = T_f(\mathbf{z}, t), \qquad (3a)$$

and

$$\mathbf{w}(\mathbf{z}, t) = \mathbf{v}(\mathbf{x}, t) - \mathbf{V}(t). \qquad (3b)$$

The energy equation for the fluid in the new coordinate system is, hence, transformed to the following:

$$\rho_f c_f \left(\frac{\partial T_f}{\partial t} + w_i \frac{\partial T_f}{\partial z_i}\right) = k_f \frac{\partial^2 T_f}{\partial z_i \partial z_i}. \qquad (4a)$$

The boundary conditions are that on the surface of the sphere the temperature of the fluid is equal to that of the sphere and that far from the sphere the temperature of the fluid approaches a given value, which is not influenced by the presence of the sphere:

$$T_f(\mathbf{z}, t) = T_s(t) \qquad \text{at } |\mathbf{z}| = \alpha$$

and

$$T_f(\mathbf{z}, t) = T_f^{\infty}(\mathbf{z}, t) \quad \text{as } |\mathbf{z}| \to \infty, \qquad (4b)$$

where T_f^{∞} is the non-uniform temperature field of the fluid far from the sphere.

For simplicity it is assumed that the thermal conductivity of the sphere is much larger than the thermal conductivity of the fluid. This implies that the Biot number for the sphere is much smaller than unity ($Bi \ll 1$). As a result any temperature gradients inside the sphere may be neglected. Hence, the temperature of the sphere is uniform in space, but still remains a function of time, $T_s(t)$. Under this condition the energy equation of the sphere is considerably simplified and may be written with respect to the rate of heat crossing its boundaries as follows:

$$m_s c_s \frac{dT_s}{dt} = -\oint_s \mathbf{q} \cdot \mathbf{n}\, dS, \tag{5}$$

where the surface integral is evaluated on the surface of the sphere. The vector $\mathbf{q}$ denotes the heat flux at the surface of the sphere and the vector $\mathbf{n}$ is the outward normal. Since the heat flux crosses from the sphere to the fluid at the boundary, the heat flux that crosses the boundary of the sphere may be written in terms of the local fluid temperature gradient:

$$\mathbf{q} = -k_f \nabla T_f(\mathbf{z}, t)_{|\mathbf{z}|=\alpha}, \tag{6a}$$

or in indicial notation:

$$q_i = -k_f \left.\frac{\partial T_f}{\partial z_i}\right|_{|\mathbf{z}|=\alpha}. \tag{6b}$$

It is apparent from equations (5), (6a) and (6b) that in order to calculate the change in the temperature of the sphere one needs the total rate of heat flow from the surface of the sphere as a function of time. This heat transfer rate will be calculated in the following sections for an arbitrary temperature field of the fluid.

CALCULATION OF THE HEAT TRANSFER RATE

Decomposition of the fluid temperature field

The method of small perturbations is used to take advantage of the small temperature field disturbance due to the presence of the sphere. Thus, the fluid temperature field is decomposed into two fields, $T_f^0(z_i, t)$, the undisturbed temperature field, which is not influenced by the presence of the sphere, and $T_f^1(z_i, t)$ the disturbance field, which is entirely due to the influence of the heat transfer from the sphere. Similarly the velocity field is decomposed into two velocities, w_i^0 the velocity of the control volume of the fluid in the absence of the sphere and w_i^1 the velocity perturbation induced by the presence of the rigid sphere. The energy equations for the two temperature fields with respect to the coordinate system moving with the sphere are:

$$\rho_f c_f \left(\frac{\partial T_f^0}{\partial t} + w_i^0 \frac{\partial T_f^0}{\partial z_i} \right) = k_f \frac{\partial^2 T_f^0}{\partial z_i \partial z_i} \tag{7}$$

and

$$\rho_f c_f \left[\frac{\partial T_f^1}{\partial t} + (w_i^0 + w_i^1) \frac{\partial T_f^1}{\partial z_i} + w_i^1 \frac{\partial T_f^0}{\partial z_i} \right] = k_f \frac{\partial^2 T_f^1}{\partial z_i \partial z_i}. \tag{8}$$

Since the undisturbed flow field may be arbitrarily imposed, its boundary conditions will not be specified.

The boundary conditions for the disturbance temperature field due to the presence of the sphere are given as follows:

$$T_f^1(\mathbf{z}, t) = T_s(t) - T_f^0(\mathbf{z}, t) \quad \text{at} \quad |\mathbf{z}| = \alpha \tag{9a}$$

and

$$T_f^1 = 0 \quad \text{at} \quad |\mathbf{z}| \to \infty. \tag{9b}$$

The calculation of the total heat transfer from the sphere rests on calculating the total heat flux from the surface of the sphere due to the two unsteady temperature fields $T_f^0(z_i, t)$ and $T_f^1(z_i, t)$. These fields are the consequence of the initial undisturbed temperature field and the disturbances the presence of the sphere produces.

Heat transfer due to the undisturbed temperature field

First the rate of heat crossing the spherical boundary due to the undisturbed temperature of the fluid will be calculated. For this purpose a control volume of the fluid is assumed, which occupies the volume of the sphere, moves with the same velocity $V_i(t)$ and its center is located at $Y_i(t)$ at all times. There is heat transfer from the control volume due to the temperature field $T_f^0(x_i, t)$, which is due to the temperature gradient of this temperature field. This rate of heat transfer may be written in terms of the temperature gradients as follows:

$$-\oint_s \mathbf{q}^0 \cdot \mathbf{n}\, dS = \oint_s k_f \nabla T_f^0 \cdot \mathbf{n}\, dS = \int_v k_f \nabla^2 T_f^0\, dV. \tag{10}$$

The last integral is evaluated inside the control volume that would have been occupied by the fluid in the absence of the sphere. In this small volume the undisturbed temperature field $T_f^0(x_i, t)$ may be approximated by a second-order Taylor expansion as follows:

$$T_f^0(\mathbf{x}, t) = T_f^0(\mathbf{Y}(t), t) + [x_i - Y_i(t)] \left.\frac{\partial T_f^0}{\partial x_i}\right|_{\mathbf{x}=\mathbf{Y}(t)} + \tfrac{1}{2}[x_i - Y_i(t)][x_j - Y_j(t)] \left.\frac{\partial^2 T_f^0}{\partial x_i \partial x_j}\right|_{\mathbf{x}=\mathbf{Y}(t)}. \tag{11}$$

The partial derivatives are to be evaluated at $Y_i(t)$, the point where the center of the sphere is present at time t. The Laplacian operator in (10) may be evaluated to yield for the rate of heat transfer due to the undisturbed flow:

$$-\oint_s \mathbf{q}^0 \cdot \mathbf{n}\, dS = \tfrac{4}{3}\pi\alpha^3 k_f \nabla^2 T_f^0 \,|_{\mathbf{x}=\mathbf{Y}(t)}, \tag{12}$$

and, since the undisturbed temperature field T_f^0 must also satisfy the energy equation,

$$k_f \nabla^2 T_f^0 = \rho_f c_f \left(\frac{\partial T_f^0}{\partial t} + u_i \frac{\partial T_f^0}{\partial x_i} \right) = \rho_f c_f \frac{DT_f^0}{Dt}, \tag{13}$$

where u_i is the undisturbed fluid velocity without the presence of the sphere in the stationary frame of reference. One may obtain by substitution of the equation (12) into (13) the rate of heat transfer due to the undisturbed temperature field:

$$-\oint_{s} \mathbf{q}^0 \cdot \mathbf{n}\,\mathrm{d}S = \tfrac{4}{3}\pi\alpha^3\rho_f c_f \frac{\mathrm{D}T_f^0}{\mathrm{D}t}\bigg|_{\mathbf{x}=\mathbf{Y}(t)}$$

$$= m_f c_f \frac{\mathrm{D}T_f^0}{\mathrm{D}t}\bigg|_{\mathbf{x}=\mathbf{Y}(t)}, \qquad (14)$$

where m_f is the mass of the fluid which would have occupied the volume of the sphere in its absence and $\mathrm{D}/\mathrm{D}t$ is the Lagrangian derivative with respect to the stationary system of coordinates $0x_1x_2x_3$. It is thus seen that the contribution of the undisturbed fluid to the heat transferred from the sphere is the same as if the sphere were a control volume occupied by the fluid itself. The total rate of heat flow from the sphere is given by the quantity in equation (14) plus the rate of heat due to the disturbance temperature field T_f^1:

$$\oint_{s} \mathbf{q}\cdot\mathbf{n}\,\mathrm{d}S = \oint_{s} \mathbf{q}^0\cdot\mathbf{n}\,\mathrm{d}S + \oint_{s} \mathbf{q}^1\cdot\mathbf{n}\,\mathrm{d}S. \qquad (15)$$

Dimensionless form of the perturbation field

The energy equation due to the disturbance caused by the presence of the sphere is given by equation (8). Given that the size of the sphere is small in comparison to the characteristic dimension of the fluid domain, it is expected that some terms in equation (8) may be much smaller than others and, hence, they should be neglected. For this reason equation (8) will be written in dimensionless form by defining the following dimensionless parameters:

$$T_f^1 = \Theta \hat{T}_f^1 \qquad (16a)$$

$$t = \tau_p \hat{t} \qquad (16b)$$

$$w_i = U\hat{w}_i \qquad (16c)$$

$$z_i = \alpha \hat{z}_i, \qquad (16d)$$

where the carets denote dimensionless quantities. U is the characteristic velocity of the system; Θ is the characteristic temperature difference of the system (e.g. the initial temperature difference between the particle and the fluid); τ_s is the characteristic time of the particle. The latter may be taken as the particle thermal response time, which is equal to $\rho_s c_s \alpha^2/k_f$. Thus, equation (8) for the disturbance field may be written in dimensionless form as follows:

$$\frac{\rho_f c_f}{\rho_s c_s}\frac{\partial \hat{T}_f^1}{\partial \hat{t}} + \frac{U\alpha\rho_f c_f}{k_f}\left[(\hat{w}_i^0 + \hat{w}_i^1)\frac{\partial \hat{T}_f^1}{\partial \hat{z}_i} + \hat{w}_i^1 \frac{\partial \hat{T}_f^0}{\partial \hat{z}_i}\right] = \frac{\partial^2 \hat{T}_f^1}{\partial \hat{z}_i\,\partial \hat{z}_i}. \qquad (17)$$

The coefficient of the second term in the above equation is the Peclet number Pe. Because of the small size of the sphere, the calculations for the temperature field due to the disturbance will be made for very small Peclet numbers. This assumption is analogous to the low Reynolds number assumption for the sphere (creeping flow assumption). The justification for this assumption lies in the fact that in creeping flow the relative velocity of the sphere with respect to the fluid, and hence the characteristic particle velocity, U, is very small. As a consequence of the low Peclet numbers, in the calculations that follow, the convective terms in the energy equation will be neglected. Thus, the creeping flow energy equation for the sphere (in dimensional form) becomes:

$$\rho_f c_f \frac{\partial T_f^1}{\partial t} = k_f \frac{\partial^2 T_f^1}{\partial z_i\,\partial z_i}. \qquad (18)$$

A symmetry relationship of temperature fields at low Peclet numbers

The heat transfer integral due to the temperature field disturbance $T_f^1(z_i, t)$, which is caused by the presence of the sphere, may be written as follows:

$$-\oint_{s} \mathbf{q}^1\cdot\mathbf{n}\,\mathrm{d}S = \oint_{s} k_f \frac{\partial T_f^1}{\partial z_i}\bigg|_{|\mathbf{z}|=\alpha} n_i\,\mathrm{d}S, \qquad (19)$$

where T_f^1 satisfies equation (18) and the boundary conditions (9a) and (9b).

It is not necessary to have a complete knowledge of the function $T_f^1(z_i, t)$ in order to obtain the heat transfer integral of equation (16). The method followed in this paper is to evaluate this integral by the use of Laplace transformations. Since T_f^1 is an unsteady field, its Laplace transform is obtained and the heat integral is evaluated in the Laplace domain. Subsequently the heat transfer integral is transformed back into the time domain to yield the complete energy equation for the sphere. This method is similar to the one used by many others in the derivation of the equation of motion of the sphere, such as Sy *et al.* [8] and Maxey and Riley [3].

In order to obtain the rate of heat transfer in the Laplace space it is necessary to derive first a symmetry relation between two temperature fields, T and T', which obey the energy equation without the convective term (according to the low Peclet number assumption). Burgers [9] derived a similar expression for the equation of motion of a particle in terms of stresses and velocities.

First consider two unsteady temperature fields $T(z_i, t)$ and $T'(z_i, t)$ in a volume V bounded by a surface S. Both of them satisfy the energy equation for the creeping flow:

$$\rho_f c_f \frac{\partial T}{\partial t} + \frac{\partial q_i}{\partial z_i} = 0, \qquad (20a)$$

with

$$q_i = -k_f \frac{\partial T}{\partial z_i}. \qquad (20b)$$

Second, by considering the volume integral of the function,

$$\int_V \left[\bar{T}\frac{\partial \bar{q}'_i}{\partial z_i} - \bar{T}'\frac{\partial \bar{q}_i}{\partial z_i}\right] dV,$$

where the overbar denotes the transformed variable in the Laplace space, and, by applying Gauss's theorem, the following symmetry relation is obtained for the two temperature fields $T(z_i, t)$ and $T'(z_i, t)$:

$$\oint_S [\bar{T}(\mathbf{z}, s)\bar{q}'_i(\mathbf{z}, s) - \bar{T}'(\mathbf{z}, s)\bar{q}_i(\mathbf{z}, s)]n_i \, dS$$

$$= \int_V \rho_i c_i [\bar{T}(\mathbf{z}, s)T'(\mathbf{z}, 0) - T(\mathbf{z}, 0)\bar{T}'(\mathbf{z}, s)] \, dV, \quad (21a)$$

where n_i is the outward normal unit vector. Under the condition that both T and T' are continuous and approach zero far from the sphere, V is the volume which includes the whole of the flow domain except the volume of the sphere and S is the surface of the sphere. In equation (21a) the parentheses enclose arguments of the functions and multiplications are denoted by the square brackets only.

This symmetry relation will be applied to the temperature field T_f^1 generated in the fluid by the disturbance caused by the sphere. The temperature field $T'(z_i, t)$ in the symmetry relation will be conveniently defined and $T(z_i, t)$ will be later identified with the disturbance field $T_f^1(z_i, t)$.

Calculation of the heat transfer due to the disturbance temperature field

The temperature field $T'(z_i, t)$ may be conveniently chosen to assist in the calculation of the heat transfer due to the field temperature $T(z_i, t)$. Here T' is chosen to satisfy the following boundary and initial conditions:

$$T'(\mathbf{z}, t \leqslant 0) = 0 \quad \text{and} \quad \bar{T}'(\mathbf{z}, s) = 1 \quad \text{on} \quad |\mathbf{z}| = \alpha. \quad (21b)$$

Thus, the temperature field T' results from a sphere at zero initial temperature which experiences an impulse of temperature, $\delta(t)$. Since the Laplace transform of T' is equal to 1, the symmetry relation (18) and the boundary–initial conditions (19) yield the following equation for the heat transfer due to the temperature field $T(z_i, t)$:

$$\oint_S \bar{q}_i n_i \, dS = \oint_S \bar{T}'\bar{q}_i n_i \, dS = \oint_S \bar{T}\bar{q}'_i n_i \, dS + \int_V \rho_f c_f \bar{T}' T_{|t=0} \, dV. \quad (21c)$$

The temperature field $T(z_i, t)$ is assigned to be equal to the disturbance temperature field $T_f^0(x_i, t)$. This field may be expressed on the surface of the sphere as a quadratic function of the coordinate system whose origin is at the center of the sphere:

$$T_f^1(\mathbf{z}, t) = A + z_i B_i + \tfrac{1}{2} z_i z_j C_{ij}, \quad \text{with} \quad T_f^1(\mathbf{z}, 0) = 0. \quad (22a)$$

The coefficients A, B_i and C_{ij} are functions of time. Their values on the surface of the sphere are obtained by a match of the disturbance temperature field with the undisturbed flow field in the interior of the sphere. A Taylor expansion of the undisturbed temperature field $T_f^0(x_i, t)$ around the center of the sphere $Y_i(t)$ yields the three coefficients of the expansion terms as follows:

$$A = T_s(t) - T_f^0(\mathbf{z} = 0, t) \quad (22b)$$

$$B_i = -\left.\frac{\partial T_f^0}{\partial x_i}\right|_{\mathbf{z}=0} \quad (22c)$$

and

$$C_{ij} = -\left.\frac{\partial^2 T_f^0}{\partial x_i \, \partial x_j}\right|_{\mathbf{z}=0}. \quad (22d)$$

The coefficients, which are known from the above expressions, are substituted into the rate of heat transfer equation from the sphere (20) to yield the following expression:

$$-\oint_S \bar{q}_i n_i \, dS = -\oint_S \bar{T}\bar{q}'_i n_i \, dS = -\oint_S (\bar{A} + \bar{B}_i z_i + \tfrac{1}{2}\bar{C}_{ij} z_i z_j)\bar{q}'_k n_k \, dS. \quad (23)$$

Thus the heat transfer due to the disturbance field may be calculated from the last two integrals in equation (23) which include the temperature T and the heat flux associated with the "auxiliary" temperature field T'. We will proceed to calculate the Laplace transform of this heat flux. It is recalled that the temperature field $T'(z_i, t)$ must also satisfy the creeping flow heat transfer equation. Hence, the following expression for the Laplace transform of $T'(z_i, t)$ is obtained:

$$s\bar{T}' - \frac{k_f}{\rho_f c_f}\frac{\partial^2 \bar{T}'}{\partial z_i \, \partial z_i} = 0, \quad (24a)$$

or in spherical coordinates,

$$s\bar{T}' - \frac{k_f}{\rho_f c_f}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \bar{T}'}{\partial r}\right) = 0, \quad (24b)$$

where s is the Laplace transform variable. Equation (24b) is rearranged to yield the following expression, which is more convenient to solve:

$$\frac{\partial^2 \bar{T}'}{\partial r^2} + \frac{2}{r}\frac{\partial \bar{T}'}{\partial r} - \lambda^2 \bar{T}' = 0, \quad (25)$$

where,

$$\lambda^2 = \frac{s}{\alpha_f} \quad \text{and} \quad \alpha_f = \frac{k_f}{\rho_f c_f}.$$

The general solution of equation (25) in the Laplace domain is easily obtained as follows:

$$\overline{T'}(r,s) = \frac{F(s)}{r}\,e^{\lambda r} + \frac{G(s)}{r}\,e^{-\lambda r}. \quad (26)$$

The Laplace transform of T' must vanish in the far field. This yields $F(s) = 0$ everywhere in the temperature domain. The quantity $G(s)$ is evaluated by the application of the second condition of equation (21b) namely that the Laplace transform of T' on the surface of the sphere is equal to one. This condition yields:

$$\overline{T'}(r,s) = \frac{\alpha}{r}\,e^{\lambda(\alpha-r)}. \quad (27)$$

Hence, the rate of heat transfer due to the temperature field T' in the Laplace domain may be obtained as follows:

$$q_i' = -k_f\frac{\partial \overline{T'}}{\partial z_i} = -k_f\alpha\left[-\frac{z_i}{r^3} - \frac{\lambda z_i}{r^2}\right]e^{\lambda(\alpha-r)}. \quad (28)$$

On the surface of the sphere, where $r = \alpha$ and $n_i = z_i/\alpha$, the last equation gives the following expression for the heat flux:

$$\overline{q_i'}(\alpha,s)n_i = k_f\frac{(1+\alpha\lambda)}{\alpha}. \quad (29)$$

Substitution of (29) into (22) yields the following relationship for the Laplace transform of the total heat transfer due to the disturbance field $T_f^1(z_i,t)$ on the surface of the sphere:

$$\bar{Q}(s) = -\oint_s \overline{q_i}n_i\,dS = -\frac{k_f(1+\lambda\alpha)}{\alpha} \times\oint_s(\bar{A}+\overline{B_i}z_i+\tfrac{1}{2}\overline{C_{ij}}z_iz_j)\,dS = -\frac{k_f(1+\lambda\alpha)}{\alpha}$$
$$\times\int_{\theta=0}^{\pi}\int_{\Phi=0}^{2\pi}(\bar{A}+\overline{B_i}z_i+\tfrac{1}{2}\overline{C_{ij}}z_iz_j)\alpha^2\sin\theta\,d\phi\,d\theta. \quad (30)$$

From its definition, C_{ij} is a symmetric tensor. The last integral in equation (30) is calculated by the expansion of its components in the spherical coordinates (r,θ,ϕ). It was observed that, because of the spherical symmetry on the surface of the sphere, the off-diagonal terms of the tensor C_{ij} and all the terms associated with the vector B_i did not contribute to the integral. Actually the following expression is obtained for the Laplace transform of the heat transfer, due to the disturbance temperature field:

$$-\oint_s \overline{q_i}n_i\,dS = -\frac{k_f(1+\lambda\alpha)}{\alpha}(4\pi\alpha^2\bar{A}+\tfrac{2}{3}\pi\alpha^3\overline{C_{ii}}). \quad (31)$$

The inverse Laplace transform of the last expression may be calculated to obtain the rate of heat transfer $Q(t)$ in the time domain due to the disturbance temperature field $T_f^1(z_i,t)$:

$$Q(t) = -\oint_s q_in_i\,dS = L^{-1}\left\{-\frac{k_f(1+\lambda\alpha)}{\alpha}\times[4\pi\alpha^2\bar{A}+\tfrac{2}{3}\pi\alpha^3C_{ii}]\right\} = -4\pi\alpha k_f[A(t)+\tfrac{1}{6}\alpha^2C_{ii}(t)]$$
$$-4\pi\alpha^2k_fL^{-1}\{\lambda[\bar{A}(s)+\tfrac{1}{6}\alpha^2\overline{C_{ii}}(s)]\}, \quad (32)$$

where λ is the quantity associated with the Laplace transform variable s [$\lambda = \sqrt{(s/\alpha_f)}$]. Since the square root of the variable s appears in the above expression one expects that a history integral will be generated in the expression for the heat transfer due to the disturbance temperature field.

The inverse Laplace transform of the last part of equation (32) is obtained by the use of the convolution theorem to yield:

$$L^{-1}[\lambda(\bar{A}+\tfrac{1}{6}\overline{C_{ii}})] = \int_0^t\frac{\frac{d}{d\tau}[A(\tau)+\frac{1}{6}\alpha^2C_{ii}(\tau)]}{[\alpha_f\pi(t-\tau)]^{1/2}}\,d\tau, \quad (33)$$

where τ is a dummy variable, which results from the use of the convolution integral and is used only in calculating the resulting history integral. The functions $A(\tau)$ and $C_{ii}(\tau)$ are obtained from the matching conditions as appear in equations (22b) and (22d):

$$A(\tau) = T_s(\tau) - T_f^0(\mathbf{z},\tau)|_{\mathbf{z}=0} \quad (34a)$$

$$C_{ii}(\tau) = -\left.\frac{\partial^2T_f^0(\mathbf{z},\tau)}{\partial z_i\,\partial z_i}\right|_{\mathbf{z}=0}. \quad (34b)$$

The heat flux associated with the disturbance field $T_f^1(z_i,t)$ was previously identified as $q_i^1(z_i,t)$. Therefore, the function of the heat transfer due to the disturbance temperature field $T_f^1(z_i,t)$ in the time domain, $Q(t)$, may be written in a final form as follows:

$$Q(t) = -\oint_s q_i^1n_i\,dS = -4\pi\alpha^2k_f\left[T_s(t)-T_f^0(\mathbf{z},t)|_{\mathbf{z}=0} - \frac{1}{6}\alpha^2\left.\frac{\partial^2T_f^0(\mathbf{z},t)}{\partial z_i\,\partial z_i}\right|_{\mathbf{z}=0}\right] - 4\pi\alpha^2k_f$$
$$\times\int_0^t\frac{\frac{d}{d\tau}\left[T_s(\tau)-T_f^0(\mathbf{z},\tau)|_{\mathbf{z}=0} - \frac{1}{6}\left.\frac{\partial^2T_f^0(\mathbf{z},\tau)}{\partial z_i\,\partial z_i}\right|_{\mathbf{z}=0}\right]}{[\pi\alpha_f(t-\tau)]^{1/2}}\,d\tau. \quad (35)$$

Energy equation for the sphere resulting from whole temperature field

The rate of heat transfer due to the undisturbed temperature field as calculated above is added to the rate of heat transfer due to the disturbance temperature field caused by the presence of the particle, to yield the total heat transfer from the sphere to the fluid as expressed in equation (15). The combination of the two rates of heat transfer leads to the following differential equation for the temperature variation of the sphere:

$$m_s c_s \frac{dT_s}{dt} = m_f c_f \frac{DT_f^0}{Dt}\bigg|_{z=0} - 4\pi\alpha k_f\left[T_s(t) - T_f^0(\mathbf{z},t)|_{z=0} - \frac{1}{6}\alpha^2 \frac{\partial^2 T_f^0(\mathbf{z},t)}{\partial z_i\,\partial z_i}\bigg|_{z=0}\right] - 4\pi\alpha^2 k_f \times \int_0^t \frac{\dfrac{d}{d\tau}\left[T_s(\tau) - T_f^0(\mathbf{z},\tau)|_{z=0} - \frac{1}{6}\alpha^2 \dfrac{\partial^2 T_f^0(\mathbf{z},\tau)}{\partial z_i\,\partial z_i}\bigg|_{z=0}\right]}{[\pi\alpha_f(t-\tau)]^{1/2}}\,d\tau. \quad (36)$$

Of the terms which appear in equation (36) the first one represents the rate of change of the temperature of the sphere. The second is the rate of heat that would have entered the control volume occupied by the sphere in the absence of the latter. This term is analogous to the "added mass" term of the equation of motion of the sphere. The third term accounts for the conduction from the sphere to the fluid due to the temperature difference and the curvature of the temperature field. The fourth term is the history integral, which appears because of the temporal as well as the spacial variation of the temperature field. This term is equivalent to the history term of the equation of motion, which is sometimes called "the Basset term". The last term accounts for the effect of all the previous temperature changes of the sphere to the current temperature change. It was found by Li and Michaelides [10] and Vojir and Michaelides [5] that this term plays an important role in the determination of the current acceleration in the equation of motion of a sphere, sometimes accounting for as much as 25% of the instantaneous acceleration. It was also found out that Boussinesq [1] obtained the same term for the equation of motion three years before Basset [2].

In order to obtain a qualitative estimate of the importance of all the terms in equation (36) a dimensionless analysis is performed. Temperatures are made dimensionless by a reference temperature Θ. The thermal response time $\tau_s = (\rho_s c_s \alpha^2)/k_f$ is used as the characteristic time of the particle. The thermal response time is used to render both t and τ dimensionless and the characteristic dimension of the undisturbed flow field L is used to make the z_i length dimensionless. The dimensionless variables are denoted by a caret. Hence, the dimensionless energy equation of the sphere becomes:

$$\frac{1}{3}\frac{d\hat{T}_s}{d\hat{t}} = \frac{1}{3}\frac{\rho_f c_f}{\rho_s c_s}\frac{D\hat{T}_f^0}{D\hat{t}}\bigg|_{z=0} - \left[\hat{T}_s - \hat{T}_f^0|_{z=0} - \frac{1}{6}\frac{\alpha^2}{L^2}\frac{\partial^2 \hat{T}_f^0}{\partial \hat{z}_i\,\partial \hat{z}_i}\bigg|_{z=0}\right] - \sqrt{\left(\frac{\rho_f c_f}{\pi\rho_s c_s}\right)} \int_0^{\hat{t}} \frac{\dfrac{d}{d\hat{\tau}}\left[\hat{T}_s - \hat{T}_f^0|_{z=0} - \dfrac{1}{6}\dfrac{\alpha^2}{L^2}\dfrac{\partial^2 \hat{T}_f^0}{\partial \hat{z}_i\,\partial \hat{z}_i}\bigg|_{z=0}\right]}{\sqrt{(\hat{t}-\hat{\tau})}}\,d\hat{\tau}. \quad (37)$$

It is apparent from equation (37) that the dimensionless variable $\eta = \rho_f c_f/\rho_s c_s$ determines the order of magnitude of the various terms. Given that the specific heats of most common solids and fluids (including gases) are of the same order of magnitude (1 kJ kg K^{-1}), then η scales as the ratio of the fluid to sphere densities. It is apparent that the first and third terms of the r.h.s. in equation (37) may be neglected in the case of gas–solid flows. However, it is expected that both of these terms will play an important role in the calculations of solid–liquid flows, where η is of the order of unity. Numerical calculations on the importance of the history term in the equation of motion of the sphere have been performed in [5 and 10]. It was observed that the history term may account for as much as 25% of the instantaneous acceleration of the sphere and that it strongly depends on the frequency of the variation of the fluid velocity. It was also found that the history term influenced the time-averaged parameters by up to 10% at low to medium frequencies. By analogy one expects similar dependence of the history term on the heat transfer from a fluid to a solid sphere, depending on the frequency of variation of the temperature domain.

It appears that the effect of the history term will be immense in the case of bubbly flows, where the ratio η is of the order of 1000. However, the authors do not wish to speculate on the importance of this term in bubbles, because some of the assumptions made in this study (e.g. low Biot number, rigid sphere) are not compatible with observations in bubbly flows.

It must be pointed out that the third component in the square brackets of the second and third terms in the r.h.s. of equation (37) scales as $(\alpha/L)^2$. This component corresponds to Oseen's correction terms in the equation of motion of the sphere. The correction terms are very small in the energy equation and may be neglected (as in the equation of motion) unless the temperature gradients are very high.

CONCLUSIONS

The unsteady heat transfer equation from a sphere moving in a viscous fluid was written in terms of the temperatures of the sphere and the fluid. The resulting temperature field was decomposed into two parts: the undisturbed unsteady temperature field in the absence of the sphere and the disturbance due to the presence of the sphere. The contribution to the change of the temperature of the sphere is obtained from the heat transfer due to the undisturbed flow and temperature field. By the use of a symmetry relationship between two temperature fields, the rate of heat transfer due to the disturbance of the temperature field caused by the presence of the sphere is first obtained in the Laplace domain and subsequently in the time domain. This rate of heat transfer reveals a history integral, which synthesizes the effects of all previous temperature changes of the sphere on the current temperature change. The term of the history integral is analogous to the history force or "Basset force", which appears in the equation of motion of the sphere. A dimensional analysis shows that this term will be of importance in liquid–solid heat transfer and by analogy in liquid–solid mass transfer processes.

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